Initial static susceptibilities of nonuniform and random Ising chains

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Abstract

Within the conventional framework of standard linear response theory we have derived exact results for the initial static susceptibilities of nonuniform spin-$\frac{1}{2}$ Ising chains. The results obtained permit one to study regularly alternating-bond and random-bond Ising chains. The influence of several types of nonuniformity and disorder on the temperature dependence of the initial longitudinal and transverse static susceptibilities is discussed.

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1 Introduction

During the last 40 years considerable effort has been devoted to the study of random magnetic systems. In order to obtain a theoretical understanding of various phenomena that can occur due to the presence of disorder it is instructive to examine simple models that allow one to calculate observable quantities exactly. Among such models, perhaps the simplest is the nonuniform spin-$\frac{1}{2}$ Ising chain specified by the Hamiltonian

$$H_0 = \sum_n J_n s_n^x s_{n+1}^x, \tag{1}$$

where the site index n of the summation takes values 1 to $N-1$, there being a total of N lattice sites. J_n is the exchange coupling between the spins at sites n and $n+1$, and s^x has eigenvalues $\pm 1/2$. The partition function of this model is readily calculated, and for open boundary conditions is [1] $Z_0 \equiv \mathrm{Tr}\,[\exp(-H_0/kT)] = 2\prod_n [2\cosh(J_n/4kT)]$, which immediately yields the entropy $S = k\,[\ln 2 + \sum_n \ln 2\cosh(J_n/4kT) - \sum_n (J_n/4kT)\tanh(J_n/4kT)]$, the specific heat $C = k\sum_n [(J_n/4kT)/\cosh(J_n/4kT)]^2$, and the correlation functions

$$\langle s_m^x s_p^x \rangle_0 = \frac{1}{4}\left(-\tanh\frac{J_m}{4kT}\right)\left(-\tanh\frac{J_{m+1}}{4kT}\right)\ldots\left(-\tanh\frac{J_{p-1}}{4kT}\right), \quad m < p. \tag{2}$$

Here, the angular brackets $\langle(\ldots)\rangle_0$ denote $(1/Z_0)\mathrm{Tr}\,[\exp(H_0/kT)(\ldots)]$. Regarding the exchange couplings in Eq. (1) as random variables having a probability distribution $p(J_1, \ldots, J_{N-1})$, one is able to study the random-averaged quantities $\overline{S}$, $\overline{C}$, or $\overline{\langle s_m^x s_p^x \rangle_0}$, defined through $\overline{Q} \equiv \int \mathrm{d}J_1 \ldots \mathrm{d}J_{N-1}\, p(J_1, \ldots, J_{N-1})Q$. In the following, attention will focus on a study of the initial (zero-field) static susceptibilities of nonuniform and random Ising chains described by Eq. (1). In the presence of a small external field h_α, $\alpha = x, y, z$, static susceptibilities may be defined by $\chi_{\beta\alpha}(h_\alpha) \equiv \partial m_\beta/\partial h_\alpha$, $m_\beta \equiv (1/N)\sum_n \langle s_n^\beta \rangle$ (in units where the product of the g-factor and the Bohr magneton is unity). The angular brackets $\langle(\ldots)\rangle$ denote thermal averages with respect to the total Hamiltonian $H = H_0 + H_\alpha$, H_α being the Zeeman term $-h_\alpha \sum_n s_n^\alpha$. The initial static susceptibilities are the limits $\chi_{\beta\alpha}(h_\alpha)]_{h_\alpha \to 0}$, to be denoted by $\chi_{\beta\alpha}$.

The static susceptibilities of random Ising chains have been studied by a number of authors. The Ising chain with random exchange coupling in a longitudinal field was considered by Fan and McCoy [2]. Assuming that each exchange coupling was an independent random variable having a probability density function with narrow width, Fan and McCoy were able to study the influence of the randomness on the longitudinal static susceptibility. The ground-state transverse static susceptibility for the random-bond Ising chain was examined by Barouch and McCoy [3]. Zaitsev studied the transverse static susceptibility for the random-bond Ising chain for various types of disorder [4]. The thermodynamics and spin correlations of a model of solid solution based on an Ising chain were considered in Ref. [5]. Some results for the initial longitudinal and transverse static susceptibilities of the random Ising chain in a transverse field were collected in Ref. [6]. Thermodynamic and dynamic properties of a random Ising chain in a transverse field for arbitrary disorder may also be examined numerically [7]. Recently, Idogaki, Rikitoku and Tucker [8] have calculated exactly the initial transverse static susceptibility for the nonuniform model of Eq. (1), and for the random version of this model in which two types of exchange couplings J_1 and J_2 are randomly distributed throughout the chain.

The object of our paper is two-fold. First, in Ref. [8] the derivation of the initial static transverse susceptibility was achieved by a somewhat unconventional technique based on an accurate extraction of the linear field-dependent term from a Callen-Suzuki identity for the magnetization. Here, we show how the result may be obtained more naturally within the framework of conventional linear response theory in which the susceptibility is related directly to the double-time correlation functions. In addition all components of the susceptibility tensor will be considered (Section 2). The second objective of this work is to apply the theory to chains having various types of disorder, to identify the extent to which qualitative features of the susceptibility are influenced by the character of the disorder, and to compare the influence of regular alternation and random disorder. Unlike, the bimodial distributions of disorder,

hitherto studied [8], more general distributions of disorder cannot be treated analytically, but require numerical computation. The result of such computations for chains having Gaussian and Lorentzian disorder are reported for the first time, in Section 3.

2 Initial static susceptibilities of nonuniform Ising chains

Our derivation of the initial static susceptibilities of a nonuniform Ising chain employs as its starting point the well-known expression

$$\chi_{\beta\alpha}(h_\alpha) = \frac{1}{kT}\frac{1}{N}\sum_{n,p}\left[\int_0^1 \mathrm{d}\tau \langle s_n^\beta\left(-\frac{\mathrm{i}\tau}{kT}\right) s_p^\alpha\rangle - \langle s_n^\beta\rangle\langle s_p^\alpha\rangle\right],$$
$$s_n^\beta(t) \equiv \mathrm{e}^{\mathrm{i}tH} s_n^\beta \mathrm{e}^{-\mathrm{i}tH}, \tag{3}$$

relating static susceptibilities to spin correlation functions. For the initial static susceptibilities ($h_\alpha \to 0$) it follows, from Eq. (3), that

$$\chi_{xx} = \frac{1}{kT}\frac{1}{N}\sum_{n,p}\langle s_n^x s_p^x\rangle_0, \tag{4}$$

and

$$\chi_{zz} = \frac{1}{kT}\frac{1}{N}\sum_{n,p}\int_0^1 \mathrm{d}\tau \langle \hat{s}_n^z\left(-\frac{\mathrm{i}\tau}{kT}\right) s_p^z\rangle_0,$$
$$\hat{s}_n^z(t) \equiv \mathrm{e}^{\mathrm{i}tH_0} s_n^\beta \mathrm{e}^{-\mathrm{i}tH_0}, \tag{5}$$

since $\langle s_n^\alpha\rangle_0 = 0$.

The integral in Eq. (5) may be effected by employing the Van der Waerden identity $\exp(\lambda s^x) = \cosh(\lambda/2) + 2\sinh(\lambda/2)s^x$. Using this identity, one finds that

$$\hat{s}_j^z(t) = \cos\left[t\left(J_{j-1}s_{j-1}^x + J_j s_{j+1}^x\right)\right] s_j^z + \sin\left[t\left(J_{j-1}s_{j-1}^x + J_j s_{j+1}^x\right)\right] s_j^y, \tag{6}$$

which, since $\cos(\lambda s^x) = \cos(\lambda/2)$ and $\sin(\lambda s^x) = 2\sin(\lambda/2)s^x$, gives

$$\hat{s}_j^z(t) = \left(\cos\frac{tJ_{j-1}}{2}\cos\frac{tJ_j}{2} - 4\sin\frac{tJ_{j-1}}{2}\sin\frac{tJ_j}{2}s_{j-1}^x s_{j+1}^x\right)s_j^z$$
$$+2\left(\sin\frac{tJ_{j-1}}{2}\cos\frac{tJ_j}{2}s_{j-1}^x + \cos\frac{tJ_{j-1}}{2}\sin\frac{tJ_j}{2}s_{j+1}^x\right)s_j^y. \tag{7}$$

With the aid of this expression, the integration in Eq. (5) is readily perform to yield the desired result

$$\chi_{zz} = \frac{1}{N}\sum_{n,p}\left[\left(\frac{\sinh\frac{J_{n-1}-J_n}{2kT}}{J_{n-1}-J_n} + \frac{\sinh\frac{J_{n-1}+J_n}{2kT}}{J_{n-1}+J_n}\right)\langle s_n^z s_p^z\rangle_0\right.$$
$$-4\left(\frac{\sinh\frac{J_{n-1}-J_n}{2kT}}{J_{n-1}-J_n} - \frac{\sinh\frac{J_{n-1}+J_n}{2kT}}{J_{n-1}+J_n}\right)\langle s_{n-1}^x s_n^z s_{n+1}^x s_p^z\rangle_0$$
$$-2\mathrm{i}\left(\frac{\cosh\frac{J_{n-1}-J_n}{2kT}-1}{J_{n-1}-J_n} + \frac{\cosh\frac{J_{n-1}+J_n}{2kT}-1}{J_{n-1}+J_n}\right)\langle s_{n-1}^x s_n^y s_p^z\rangle_0$$
$$\left.+2\mathrm{i}\left(\frac{\cosh\frac{J_{n-1}-J_n}{2kT}-1}{J_{n-1}-J_n} - \frac{\cosh\frac{J_{n-1}+J_n}{2kT}-1}{J_{n-1}+J_n}\right)\langle s_n^y s_{n+1}^x s_p^z\rangle_0\right]. \tag{8}$$

Since H_0 does not contain s^y or s^z, and $s^y s^z = (\mathrm{i}/2)s^x$, $\mathrm{Tr}s^\alpha = 0$ and $(s^\alpha)^2 = 1/4$, it is readily seen on using a representation in which s^x is diagonal that the correlation functions appearing in Eq. (8) can be simplified. Namely, $\langle s_n^z s_p^z\rangle_0 = (1/4)\delta_{np}$, $\langle s_{n-1}^x s_n^z s_{n+1}^x s_p^z\rangle_0 = (1/4)\delta_{np}\langle s_{n-1}^x s_{n+1}^x\rangle_0$, $\langle s_{n-1}^x s_n^y s_p^z\rangle_0 = (\mathrm{i}/2)\delta_{np}\langle s_{n-1}^x s_n^x\rangle_0$, $\langle s_n^y s_{n+1}^x s_p^z\rangle_0 = (\mathrm{i}/2)\delta_{np}\langle s_n^x s_{n+1}^x\rangle_0$. The pair correlation functions in s^x that remain may then be evaluted using Eq. (2). In this way, it follows after straightforward calculations, that the initial longitudinal and transverse static susceptibilities, Eq. (4) and Eq. (8), of the nonuniform Ising chain may be expressed in the form

$$\chi_{xx} = \frac{1}{4kT}\left[1 + \frac{2}{N}\sum_n\sum_{p>n}\left(-\tanh\frac{J_n}{4kT}\right)\left(-\tanh\frac{J_{n+1}}{4kT}\right)\ldots\left(-\tanh\frac{J_{p-1}}{4kT}\right)\right]; \tag{9}$$

$$\chi_{zz} = \frac{1}{2N}\sum_n\left(\frac{\tanh\frac{J_{n-1}}{4kT} - \tanh\frac{J_n}{4kT}}{J_{n-1}-J_n} + \frac{\tanh\frac{J_{n-1}}{4kT} + \tanh\frac{J_n}{4kT}}{J_{n-1}+J_n}\right). \tag{10}$$

The result, Eq. (10), for χ_{zz} coincides with the result obtained by Idogaki, Rikitoku and Tucker [8] using a different approach.

The off-diagonal components $\chi_{\alpha\beta}$ can be shown to be zero as follows. $\chi_{\beta\alpha}$ contains $\mathrm{Tr}[\exp(-H_0/kT)\hat{s}^\beta(t)s^\alpha]$. Again, using a representation in which s^x is diagonal, this trace

is zero if $\beta = x$ and $\alpha = y$ or z. Thus χ_{xy} and χ_{xz} are zero. Also, from the definition of the interaction representation, $\hat{s}(t)$, and the fact that a trace is invarient under cyclic rotation of the operators under it, it follows that χ_{yx} and χ_{zx} are also zero. Further, χ_{zy} is given by Eq. (8) with s_p^z replaced by s_p^y. All the correlation functions on the right hand side then reduce to zero, so $\chi_{zy} = 0$. Finally, $\chi_{yz} = \chi_{zy}$ $(= 0)$ and $\chi_{yy} = \chi_{zz}$ from the symmetry transformation $s^{x\prime} = -s^x$, $s^{y\prime} = s^z$, $s^{z\prime} = s^y$.

One immediately observes for the regular chain, $J_1 = J_2 = \dots$, that Eq. (9) gives [1, 9]

$$\chi_{xx}^{J_1} = \frac{1}{4kT}\left[1 + \frac{2}{N}\sum_q (N-q)\left(-\tanh\frac{J_1}{4kT}\right)^q\right] = \frac{1}{4kT}\left(1 - \frac{2\tanh\frac{J_1}{4kT}}{1+\tanh\frac{J_1}{4kT}}\right) = \frac{\mathrm{e}^{-\frac{J_1}{2kT}}}{4kT}, \tag{11}$$

in the thermodynamic limit. For the regular alternating-bond chain, $J_1 = J_3 = \dots$, $J_2 = J_4 = \dots$, Eq. (9) yields

$$\chi_{xx}^{J_1 J_2} = \frac{1}{4kT}\frac{\left(1-\tanh\frac{J_1}{4kT}\right)\left(1-\tanh\frac{J_2}{4kT}\right)}{1-\tanh\frac{J_1}{4kT}\tanh\frac{J_2}{4kT}}, \tag{12}$$

and for the case of the regular alternating-bond chain $J_1 = J_2 = J_5 = J_6 \dots$, $J_3 = J_4 = J_7 = J_8 \dots$

$$\chi_{xx}^{J_1 J_1 J_3 J_3} = \frac{1}{4kT}\left[1 - \frac{\tanh\frac{J_1}{4kT} + \tanh\frac{J_3}{4kT}}{1-\tanh\frac{J_1}{4kT}\tanh\frac{J_3}{4kT}} + \frac{\frac{1}{2}\left(\tanh\frac{J_1}{4kT}+\tanh\frac{J_3}{4kT}\right)^2 + 2\tanh^2\frac{J_1}{4kT}\tanh^2\frac{J_3}{4kT}}{1-\tanh^2\frac{J_1}{4kT}\tanh^2\frac{J_3}{4kT}}\right]. \tag{13}$$

Further, for the transverse components, Eq. (10) yields the results: $\chi_{zz}^{J_1}$ [10], $\chi_{zz}^{J_1 J_2}$ [8], $\chi_{zz}^{J_1 J_1 J_3 J_3} = \frac{1}{4}\chi_{zz}^{J_1} + \frac{1}{4}\chi_{zz}^{J_3} + \frac{1}{2}\chi_{zz}^{J_1 J_3}$, $\chi_{zz}^{J_1 J_1 J_1 J_4 J_4 J_4} = \frac{1}{3}\chi_{zz}^{J_1} + \frac{1}{3}\chi_{zz}^{J_4} + \frac{1}{3}\chi_{zz}^{J_1 J_4}$ etc..

It is worthwhile to note, that χ_{zz}, Eq. (10), does not depend on the sign of the intersite coupling, whereas χ_{xx}, Eq. (9), does. Thus, according to Eq. (11) for the regular chain, one finds immediately that for ferromagnetic coupling $(J_1 < 0)$ $\chi_{xx} \to \infty$ as $T \to 0$, whereas for antiferromagnetic coupling $(J_1 > 0)$ $\chi_{xx} \to 0$ as $T \to 0$. In the case of the regular alternating-bond chain $J_1 = J_3 = \dots$, $J_2 = J_4 = \dots$, Eq. (12) implies that χ_{xx} diverges as $T \to 0$

if both J_1 and J_2 are ferromagnetic, whereas if either, or both, of the intersite couplings is antiferromagnetic, it does not diverge. From Eq. (13) for the alternating-bond chain $J_1 = J_2 = J_5 = J_6 = \ldots, J_3 = J_4 = J_7 = J_8 = \ldots$, one finds that if both, or either, of J_1 and J_3 are ferromagnetic χ_{xx} diverges as $T \to 0$, whereas, it does not, if both couplings are antiferromagnetic. These results are to be expected if one considers the ground state of these regular alternating-bond spin chains. For example, it is clear that the ground state of the chain $J_1 = J_3 = \ldots, J_2 = J_4 = \ldots$, with $J_1 < 0, J_2 > 0$, is of an antiferromagnetic type, whereas the ground state of the chain $J_1 = J_2 = J_5 = J_6 = \ldots, J_3 = J_4 = J_7 = J_8 = \ldots$, having $J_1 < 0, J_3 > 0$, is ferromagnetic in character.

3 Initial static susceptibilities of random Ising chains

Consider an Ising chain in which the exchange couplings are random variables evenly distributed with a probability $p(J_1, \ldots, J_{N-1}) = \prod_n p(J_n)$. The random-averaged susceptibilities are given by

$$\overline{\chi_{xx}} = \frac{1}{4kT}\left[1 + \frac{2}{N}\sum_q (N-q)\overline{\left(-\tanh\frac{J}{4kT}\right)}^q\right] \\ = \frac{1}{4kT}\left(1 - \frac{2\overline{\tanh\frac{J}{4kT}}}{1 + \overline{\tanh\frac{J}{4kT}}}\right) \tag{14}$$

(in the first equality use has been made of the fact that $\overline{-\tanh(J_p/4kT)}$ is the same for all sites p), and

$$\overline{\chi_{zz}} = \frac{1}{2}\left[\overline{\left(\frac{\tanh\frac{J_1}{4kT} - \tanh\frac{J_2}{4kT}}{J_1 - J_2}\right)} + \overline{\left(\frac{\tanh\frac{J_1}{4kT} + \tanh\frac{J_2}{4kT}}{J_1 + J_2}\right)}\right]. \tag{15}$$

Besides the probability distribution

$$p(J_n) = c\delta(J_n - J_1) + (1-c)\delta(J_n - J_2), \quad 0 \le c \le 1 \tag{16}$$

for which $\overline{\chi_{zz}}$ was examined in Ref. [8], the Gaussian distribution

$$p(J_n) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(J_n - J_0)^2}{2\sigma^2}\right] \tag{17}$$

and the Lorentzian distribution

$$p(J_n) = \frac{1}{\pi} \frac{\Gamma}{(J_n - J_0)^2 + \Gamma^2} \tag{18}$$

centred at J_0 with a strength of disorder controlled by σ^2 and Γ, respectively, will be considered.

Consider first the longitudinal susceptibility $\overline{\chi_{xx}}$, Eq. (14), (Figs. 1-3). The low-temperature behaviour, discussed in Section 2, for the uniform chain arose because $\tanh(J/4kT) \to \mathrm{sgn} J$ as $T \to 0$. As a result, in this limit χ_{xx} diverges more rapidly than $1/T$ for $J < 0$, but does not diverge for $J > 0$. For the random case, the essential quantity influencing the initial longitudinal static susceptibility is $\mid \overline{\tanh(J/4kT)} \mid$. If there are present, exchange interactions of opposite sign, then this quantity is < 1 as $T \to 0$ and a qualitative change in the temperature dependence of χ_{xx} from that of the uniform chain results. The low temperature behaviour is now of the paramagnetic type, $1/T$, (the dotted lines in Figs. 1b, 1c and 2). A similar qualitative change in the temperature dependence occurs in the presence of randomness defined by Eqs. (17) or (18) (the dotted lines in Fig. 3). On the other hand, in the presence of any randomness defined by Eq. (16) in which J_1 and J_2 have the same sign, the quantity $\mid \overline{\tanh(J/4kT)} \mid$ still equals unity in the limit $T \to 0$, and thus this randomness only leads to a quantitative change in the thermal dependence of $\overline{\chi_{xx}}$ from that of the uniform case (the dotted lines in Figs. 1a, 1d). In Figs. 2 and 3 one can also trace a crossover from a ferromagnetic or antiferromagnetic type of thermal behaviour at high temperatures to a paramagnetic type of thermal behaviour as $T \to 0$, which occurs with the introduction of random-bond disorder given by distributions of Eqs. (16) with $J_1 J_2 < 0$, (17) or (18). Any small amount of disorder causes this crossover, but the smaller the disorder the lower is the temperatures at which it occurs.

It is also of interest to compare the results for the alternating-bond chains of Section 2 with that for the random chain (described by Eq. (16)) having equal concentrations of the two types of bond (i.e. $c = 0.5$). It was seen for the regular alternating-bond models having $J_1 > 0$ and $J_2 < 0$ (or vice versa), that the low temperature longitudinal susceptibility either did (in the case of $J_1 J_1 J_2 J_2 \ldots$), or did not (in the case of $J_1 J_2 J_1 J_2 \ldots$) diverge at low temperatures. Further, the divergence when it occurs, is greater than $1/T$. For the corresponding random chain ($c = 0.5$) having these signs for the exchange interactions, it has been observed above that a $1/T$ dependence results instead. (Compare the short-dashed and long-dashed lines with the curve for $c = 0.5$ in Figs. 1b, 1c and 2). On the other hand, when J_1 and J_2 are either both positive or negative, the corresponding curves only differ quantitatively (Figs. 1a and 1d).

Finally, it is noted that the initial longitudinal static susceptibility for chains with Gaussian and Lorentzian disorders having $J_0 = 0$ exhibit the temperature dependence of an ideal paramagnetic $\chi_{xx} = 1/4kT$, independent of the strength of the disorder.

Consider now the thermal behaviour of the initial transverse static susceptibility $\overline{\chi_{zz}}$, Eq. (15), (Figs. 4-7). For the uniform Ising chain it is finite at all temperatures; equal to $1/(2J)$ at $T = 0$, slightly increasing with increase of temperature, and then decreasing to approach the temperature dependence characteristic of an ideal paramagnet at high temperatures. The weaker the exchange interaction between the longitudinal (x) components of the spins, the larger is the value of χ_{zz} at zero temperature, the smaller is the temperature at which χ_{zz} exhibits its maximum, and the smaller is the temperature at which it exhibits behaviour characteristic of an ideal paramagnet. These features will be reflected in the sequence of curves for the temperature dependence of $\overline{\chi_{zz}}$ of the random chain, described by Eq. (16), as the concentration is varied. Fig. 4 shows such a series of curves as c varies from 1 to 0 for the particular case when $J_1 = \mid J \mid$ and $J_2 = 0.3 \mid J \mid$. The susceptibilities of some regular alternating-bond chains are also depicted. It is interesting to note that the initial transverse static susceptibility for

the random-bond chain having $c = 0.5$ is enhanced over that for the alternating-bond chain $J_1J_2J_1J_2\dots$, coincides with that for the chain $J_1J_1J_2J_2J_1J_1J_2J_2\dots$, and is suppressed in comparison to that for the chain $J_1J_1J_1J_2J_2J_2\dots$, etc..

Figs. 5,6 illustrate a difference in the influence of Gaussian, Eq. (17), and Lorentzian, Eq. (18), bond disorder on the temperature dependence of $\overline{\chi_{zz}}$. For increasing, but small, Gaussian disorder strength an increasing enhancement of $\overline{\chi_{zz}}$ occurs at low temperatures (curves 2-4 in Fig. 5). Above a certain disorder strength, this low temperature enhancement is reduced (curves 5,6), and finally a suppression of $\overline{\chi_{zz}}$ below its non-disordered value occurs (curve 7). In contrast to this behaviour for Gaussian disorder, chains with Lorentzian disorder exhibit a decrease in $\overline{\chi_{zz}}$ with increasing disorder strength for almost all temperatures (Fig. 6).

Finally, it is noted that in contrast to the initial longitudinal static susceptibility, the initial transverse static susceptibility for chains having Gaussian or Lorentzian disorder, with $J_0 = 0$, decreases with increase of disorder strength (Fig. 7). This is not surprising, since the larger the exchange coupling, the smaller is χ_{zz}, and with increasing σ^2 or Γ larger values of exchange couplings are present more often.

4 Discussion

To summarise. An alternative, but more natural derivation than that of Ref. [8], of the exact formulae for the initial static susceptibility of random-bond Ising chains has been presented. The former requires evaluation of the thermal averaged magnetization to first-order in the magnetic field strength, whereas the latter, requires evaluation of just the field-independent part of the double-time pair correlation function. The latter approach also permits one to derive similarly the frequency dependent susceptibility and the structure factor of nonuniform and random spin-$\frac{1}{2}$ Ising chains; work in this direction is in progress. Dependent on the type of

randomness present, it has been demonstrated that the disorder may lead to either a qualitative, or to only a quantitative change, in the thermal dependence of the initial static susceptibilities. Even a small amount of disorder may lead to a qualitative deviation from that of the nonrandom case at low temperatures. A difference in the temperature dependence of the initial static susceptibilities of regular alternating-bond chains having two types of exchange bonds, and the corresponding random-bond chain, has been revealed. Comparison of the numerical results for chains having Gaussian bond-disorder and for those having Lorentzian bond-disorder show a marked difference in the dependence of the transverse susceptibility on the strength of the disorder. Although, the theoretical results observed in our work should prove valuable in understanding the effects of disorder on the observable properties of Ising chain materials, there are to our knowledge, no experimental results yet available that enable a direct comparison between theory and experiment to be made. However, with for example, the synthesis of magnetic chain molecular materials becoming a reality, this lack of experimental data may be resolved in the future.

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Fig. 1. Thermal dependence of the initial longitudinal static susceptibility of disordered-bond (described by Eq. (16)) and regular alternating-bond Ising chains when J_1 and J_2 are respectively, (a) $-\mid J\mid$, $-0.3\mid J\mid$, (b) $-\mid J\mid$, $0.3\mid J\mid$, (c) $\mid J\mid$, $-0.3\mid J\mid$ and (d) $\mid J\mid$, $0.3\mid J\mid$. Curve 1: uniform chain, $c=1$. Curves 2 and 3: regular alternating chains $J_1J_2J_1J_2\ldots$, $J_1J_1J_2J_2\ldots$, respectively. Curve 4: disordered chain, $c=0.5$. Curve 5: uniform chain, $c=0$. Curve 6: ideal paramagnet, $J_1=J_2=0$.

Fig. 2. Thermal dependence of $kT\overline{\chi_{xx}}$ for disordered-bond (described by Eq. (16)) and regular alternating-bond Ising chains when $J_1=-\mid J\mid$ and $J_2=0.3\mid J\mid$. Curve 1: uniform chain, $c=1$. Curves 2-8: disordered chains, $c=0.9$, 0.8, 0.7, 0.5, 0.3, 0.2 and 0.1, respectively. Curve 9: uniform chain $c=0$. Curves 10 and 11: regular alternating-bond chains $J_1J_2J_1J_2\ldots$ and $J_1J_1J_2J_2\ldots$, respectively. Curve 12: ideal paramagnet, $J_1=J_2=0$.

Fig. 3. Thermal dependence of $kT\overline{\chi_{xx}}$ for Ising chains with (a) Gaussian and (b) Lorentzian bond disorder. Curves 1 to 3 are for $J_0=-\mid J\mid$ with $(\sigma/\mid J\mid)^2=0$ or $\Gamma/\mid J\mid=0$, $(\sigma/\mid J\mid)^2=0.5$ or $\Gamma/\mid J\mid=0.5$ and $(\sigma/\mid J\mid)^2=1$ or $\Gamma/\mid J\mid=1$, as appropriate to (a) and (b), respectively. Curves 4 to 6 are for $J_0=\mid J\mid$, with the same pairs of $(\sigma/\mid J\mid)^2$ and $\Gamma/\mid J\mid$ as for curves 1 to 3, respectively. Curve 7 is for the ideal paramagnet, $J_0=0$.

Fig. 4. Thermal dependence of the initial transverse static susceptibility of disordered-bond (described by Eq. (16)) and regular alternating-bond Ising chains when $J_1=\mid J\mid$ and $J_2=0.3\mid J\mid$. Curve 1: uniform chain, $c=1$. Curves 2 to 4: disordered chains, $c=0.7$, 0.5, 0.3, respectively. Curve 5: uniform chain $c=0$. Curves 6 to 8: regular alternating-bond chains $J_1J_2J_1J_2\ldots$, $J_1J_1J_2J_2\ldots$ and $J_1J_1J_1J_2J_2J_2\ldots$, respectively.

Fig. 5. Thermal dependence of the initial transverse static susceptibility of Ising chains with Gaussian bond-disorder, when $J_0 =\mid J \mid$. Curve 1: no disorder, $(\sigma/\mid J \mid)^2 = 0$. Curves 2 to 7: $(\sigma/\mid J \mid)^2 = 0.1$, 0.2, 0.5, 1, 2 and 4, respectively.

Fig. 6. Thermal dependence of the initial transverse susceptibility of Ising chains with Lorentzian bond-disorder, when $J_0 =\mid J \mid$. Curve 1: no disorder, $\Gamma/\mid J \mid= 0$. Curves 2-6, $\Gamma/\mid J \mid= 0.02$, 0.1, 0.2, 0.5 and 1, respectively.

Fig. 7. Thermal dependence of the initial transverse susceptibility of Ising chains with (a) Gaussian and (b) Lorentzian bond-disorder, when $J_0 = 0$. Curve 1: ideal paramagnet, $(\sigma/\mid J \mid)^2 = 0$ or $\Gamma/\mid J \mid= 0$, as appropriate in (a) and (b). Curves 2 and 3: $(\sigma/\mid J \mid)^2 = 0.5$ or $\Gamma/\mid J \mid= 0.5$ and $(\sigma/\mid J \mid)^2 = 1$ or $\Gamma/\mid J \mid= 1$, respectively.